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NUMERICAL METHODS FOR SINGULARLY PERTURBED DIFFERENTIAL
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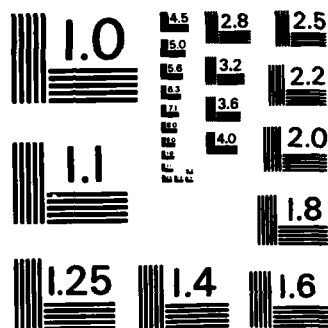
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INTERIM SCIENTIFIC REPORT

Air Force Office of Scientific Research Grant AFOSR-80-0192

Period: 1 June 1981 through 31 May 1982

Title of Research: Numerical Methods for Singularly
Perturbed Differential Equations
with Applications

Principal Investigator: Joseph E. Flaherty

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ABSTRACT

During the period covered by this report we continued our research on the development and application of numerical methods for singularly-perturbed (or stiff) boundary value problems for ordinary differential equations and initial-boundary value problems for partial differential equations. Results were obtained for collocation methods for vector systems of two-point boundary value problems and for adaptive grid finite element methods for partial differential equations. We are applying our methods to several interesting physical problems, such as, the deformation of nonlinear elastic beams and a nonlinear Schrodinger equation which exhibits self focusing.

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MATTHEW J. KERPER
Chief, Technical Information Division

1. Progress and Status of the Research on Numerical Methods for Singularly Perturbed Differential Equations

During the period covered by this report we continued our research on the development and application of numerical methods for singularly-perturbed ordinary and partial differential equations.

1.1 Boundary Value Problems for Ordinary Differential Equations

Our work on combined asymptotic and numerical (collocation) techniques for vector systems of boundary value problems is continuing and it will appear in a sequence of two papers [4,5]*. A third paper [7] on the subject is being prepared for the proceedings of the Tenth IMACS World Conference, which will be held in Montreal this August, and a fourth paper [8] will be submitted to SIAM Journal of Scientific and Statistical Computing later this year. All of these papers are joint research between Professors J. E. Flaherty and R. E. O'Malley, Jr. Professor O'Malley joined R.P.I. as Chairman of the Department of Mathematical Sciences in June 1981.

We have applied our methods to some interesting and challenging nonlinear singularly-perturbed problems and have found them to be both efficient and accurate. One area where our methods seem to be very successful is in studying the deformation of nonlinear elastic beams (cf. [4]).

Flaherty, O'Malley, and G. M. Heitker, a graduate student under their direction, have recently begun studying collocation and difference methods for vector systems of boundary value problems that have potential for turning point problems. In the spirit of our earlier work on scalar equations**, these methods have the advantage of only needing a fine discretization in

*See the list of Publications and Abstracts at the end of this report.

**J. E. Flaherty and W. Mathon, "Collocation with Polynomial and Tension Splines for Singularly Perturbed Boundary Value Problems, SIAM J. Sci. and Stat. Comput., 1 pp. 260-289 (1980).

boundary layer regions for accuracy and not for stability. Thus, we can create an algorithm that starts with a relatively coarse discretization to obtain a stable solution and then adaptively refines the grid within boundary layer regions for greater accuracy.

1.2. Initial-Boundary Value Problems for Partial Differential Equations

A paper [1] describing our adaptive grid finite element procedure for initial-boundary value problems for vector systems of partial differential equations has appeared in the SIAM Journal on Scientific and Statistical Computing. This work was based on the Ph.D. Dissertation of S. F. Davis who was a graduate scholar under the direction of Professor Flaherty and who was supported by this grant.

Flaherty and a graduate student, J. M. Coyle, have added several improvements to our adaptive finite element code. These include time step refinement routines and the ability to add or delete elements. They have also begun work on a two-dimensional version of the code.

Flaherty, Coyle, and A. C. Newell of the University of Arizona have been applying our adaptive finite element code to a focusing problem for a non-linear Schrodinger equation [9]. This problem describes the focusing of a laser beam in a medium with a nonlinear index of refraction. It is a difficult numerical problem because the amplitude of the solution becomes infinite as focusing occurs; however, our adaptive code seems to be able to cope with this and is providing very good results.

Our work on using splines in tension to construct explicit finite difference and finite element schemes for hyperbolic and parabolic systems of partial differential equations has appeared in the Transactions of the Twenty-Seventh Conference of Army Mathematicians [2]. A second note on this subject

will appear in the International Journal for Numerical Methods in Engineering [3].

Flaherty and T. Jackson, a graduate student, have recently begun working on constructing implicit discontinuous finite element methods for hyperbolic systems. The discontinuous finite elements appear to be giving very sharp shocks, with no spurious oscillations or diffusion and without the need to explicitly track the shocks.

This grant provides funds for a visitor, and this year we invited David Gottlieb of ICASE, NASA Langley Research Center, to R.P.I. He gave a lecture on pseudo-spectral methods for hyperbolic problems with shock waves. We also held several informal discussions on problems of mutual interest.

2. Interactions

Professor Flaherty was invited to lecture on material pertaining to this grant at the following conferences or organizations:

Twenty-Seventh Conference of Army Mathematicians, USMA, West Point,
10-12 June, 1981.

Courant Institute of Mathematical Sciences, New York University,
11 December, 1981.

1982 Army Numerical Analysis and Computers Conference, WES,
Vicksburg, MS, 2-4 February 1982.

Program in Applied Mathematics, University of Arizona, Tucson, AZ
(gave a series of lectures on adaptive methods for partial differential equations).

Professor Flaherty visited the following laboratories and organizations and conducted or discussed research on the topics noted:

ICASE, NASA Langley Research Center, Hampton, Virginia, 17-24 August, 1981 and 24-28 May, 1982. Discussed and performed research on adaptive grid finite element methods and numerical methods for singularly-perturbed boundary value problems.

Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, New York, one day per week. Conducted research on adaptive grid and exponentially weighted finite element methods. Applied these methods to wave propagation, impact, and penetration problems.

Organized with Jagdish Chandra of the U. S. Army Research Office the "ARO Workshop on Computational Aspects of Penetration Mechanics", held 27-29 April, 1982 at Aberdeen Proving Ground, MD.

3. List of publications and Manuscripts in Preparation

Publications

1. S. F. Davis and J. E. Flaherty, "An Adaptive Finite Element Method for Initial-Boundary Value Problems", SIAM J. Sci. and Stat. Comp., Vol. 3, No. 1 (1982), pp. 6-17.
2. J. E. Flaherty, "Explicit Difference Schemes for Wave Propagation and Impact Problems", Trans. Twenty-seventh Conf. of Army Mathematicians, ARO Rep. 82-1 (1982), pp. 321-337.

In Press

3. J. E. Flaherty, "A Rational Function Approximation for the Integration Point in Exponentially Weighted Finite Element Methods", Tech. Rep. ARLCB-TR-81022, Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, New York, June, 1981. Also, accepted for publication in Int. J. Num. Meth. Engng., June, 1981.
4. J. E. Flaherty and R. E. O'Malley, Jr., "Singularly Perturbed Boundary Value Problems for Nonlinear Systems, Including a Challenging Problem for a Nonlinear Beam", to appear in W. Eckhaus and E. H. deJager (Eds.), Proc. Conf. on Singular Perturbations and Applics., Lect. Notes in Maths., Springer-Verlag, Berlin, 1982.
5. J. E. Flaherty and R. E. O'Malley, Jr., "Asymptotic and Numerical Methods for Vector Systems of Singularly-Perturbed Boundary Value Problems, to appear in Trans. Army Numer. Anal. and Comp. Conf., Vicksburg, MS, Feb. 1982.

6. C. Kleinstreuer, A. Eklina, and J. E. Flaherty, "New Higher-Order Boundary-Layer Equations for Laminar and Turbulent Flow past Axisymmetric Bodies", submitted to SIAM J. Appl. Math. April, 1982.

In preparation

7. R. E. O'Malley, Jr. and J. E. Flaherty, "On the Numerical Solution of Singularly-Perturbed Boundary Value Problems", in preparation for Trans. IMACS Conf., Montreal, Que., August 1982.
8. R. E. O'Malley, Jr. and J. E. Flaherty, "Numerical Methods for Stiff Systems of Two-Point Boundary Value Problems", in preparation for an ICASE Report, NASA Langley Research Center, Hampton, VA and SIAM J. Sci. and Stat. Comput.
9. J. M. Coyle, J. E. Flaherty, and A. C. Newell, "Focusing Problems for Damped and Undamped Nonlinear Schrodinger Equation", in preparation for Physica D.

AN ADAPTIVE FINITE ELEMENT METHOD FOR INITIAL-BOUNDARY
VALUE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS

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and

Joseph E. Flaherty
Rensselaer Polytechnic Institute
and
Institute for Computer Applications
in Science and Engineering

ABSTRACT

A finite element method is developed to solve initial-boundary value problems for vector systems of partial differential equations in one space dimension and time. The method automatically adapts the computational mesh as the solution progresses in time and is thus able to follow and resolve relatively sharp transitions such as mild boundary layers, shock layers, or wave fronts. This permits an accurate solution to be calculated with fewer mesh points than would be necessary with a uniform mesh.

The overall method contains two parts, a solution algorithm and a mesh selection algorithm. The solution algorithm is a finite element-Galerkin method on trapezoidal space-time elements, using either piecewise linear or cubic polynomial approximations and the mesh selection algorithm builds upon similar work for variable knot spline interpolation.

A computer code implementing these algorithms has been written and applied to a number of problems. These computations confirm that the theoretical error estimates are attained and demonstrate the utility of variable mesh methods for partial differential equations.

EXPLICIT DIFFERENCE SCHEMES FOR WAVE
PROPAGATION AND IMPACT PROBLEMS

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ABSTRACT

Explicit finite difference and finite element schemes are constructed to solve propagation, shock, and impact problems. The schemes are of upwind difference type, but suffer less from the effects of numerical dispersion and diffusion than classical upwind schemes. The relationship of the new schemes to existing explicit schemes is analyzed and numerical results and comparisons are presented for several examples.

A RATIONAL FUNCTION APPROXIMATION
FOR THE INTEGRATION POINT
IN EXPONENTIALLY WEIGHTED FINITE ELEMENT METHODS

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ABSTRACT

A rational function is presented for approximating the function $f(z) = \coth z - 1/z$ that appears in several exponentially fitted or weighted finite difference and finite element methods for convection-diffusion problems. The approximation is less expensive to evaluate than $f(z)$ and provides greater accuracy than the doubly asymptotic approximation when $z = O(1)$.

Tech. Rep. ARLCB-TR-81022, Benet Weapons Laboratory, Watervliet Arsenal,
Watervliet, New York, June, 1981. Also accepted for publication in INT. J.
Num. Math. Engng. June 1981.

SINGULARLY PERTURBED BOUNDARY VALUE PROBLEMS
FOR NONLINEAR SYSTEMS, INCLUDING A CHALLENGING PROBLEM
FOR A NONLINEAR BEAM

by

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ABSTRACT

We consider certain singularly perturbed two-point boundary value problems involving nonlinear vector systems

$$\dot{x} = f(x, y, t, \epsilon), \quad \epsilon \dot{y} = g(x, y, t, \epsilon)$$

of $m + n$ ordinary differential equations on a finite interval $0 < t < 1$ subject to q initial conditions and r terminal conditions of the form

$$A(x(0), y(0), \epsilon) = 0, \quad B(x(1), y(1), \epsilon) = 0,$$

with $q + r = m + n$. Most critically, in addition to natural smoothness assumptions, we assume that the $n \times n$ Jacobian matrix $g_y(x, y, t, 0)$ has a hyperbolic splitting with $k > 0$ stable eigenvalues (i.e., eigenvalues having strictly negative real parts) and $n - k > 0$ (strictly) unstable eigenvalues for all x and y and $0 < t < 1$. We suppose that $q > k$ and $r > n - k$ and we find limiting solutions as the small positive parameter ϵ tends to zero.

We apply our asymptotic methods to study the deformation and stresses in a thin nonlinear elastic beam resting on a nonlinear elastic foundation. Results are presented for simple, clamped, and elastic support conditions.

To appear in W. Eckhaus and E. H. deJager (Eds.), Proc. Conf. on Singular Perturbations and Applic., Lect. Notes in Maths., Springer-Verlag, Berlin, 1982.

ASYMPTOTIC AND NUMERICAL METHODS FOR VECTOR SYSTEMS
OF SINGULARLY-PERTURBED BOUNDARY VALUE PROBLEMS

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ABSTRACT

Procedures are developed for constructing asymptotic solutions for certain nonlinear singularly-perturbed vector two-point boundary value problems having boundary layers at one or both end points. The asymptotic approximations are generated numerically and can either be used as is or to furnish a two-point boundary value code (e.g. COLSYS) with an initial approximation and a nonuniform computational mesh. The procedures are applied to several examples involving the deformation of nonlinear elastic beams.

To appear in Trans. Army Numer. Anal. and Comp. Conf., Vicksburg, MS, Feb. 1982.

NEW HIGHER-ORDER BOUNDARY-LAYER EQUATIONS
FOR LAMINAR AND TURBULENT FLOW PAST AXISYMMETRIC BODIES

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ABSTRACT

New sets of boundary-layer equations accounting for flow field non-uniformities such as curvature effects, normal stress and pressure variations as well as separation, are derived. The boundary-layer flow domain is subdivided into (1) a parabolic region where the fluid flow is approximately parallel to the submerged body, i.e. $v \ll u$ and (2) an elliptic region which includes the line of separation where significant interactions between the boundary-layer and the outer potential flow occur, i.e. $v \approx u$. Closure for the turbulent flow equations has to be obtained with submodels for the Reynolds stresses which reflect the effects of boundary-layer thickening as well as separation. The accuracy of the parabolic equations was compared with Van Dyke's higher-order boundary-layer equations for laminar flow past a body with longitudinal curvature. The results demonstrate that the new modeling equations make a measurable difference as expected from observations made by Bradshaw and others.

Submitted to SIAM J. Appl. Maths., April, 1982.

ON THE NUMERICAL SOLUTION OF SINGULARLY-PERTURBED VECTOR
BOUNDARY VALUE PROBLEMS

by

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ABSTRACT

Numerical procedures are developed for constructing asymptotic solutions of certain nonlinear singularly-perturbed vector two-point boundary value problems having boundary layers at one or both end points. The asymptotic approximations are generated numerically and can either be used as is or to furnish a two-point boundary value code (e.g. COLSYS) with an initial approximation and a nonuniform computational mesh. The procedures are applied to a model problem that indicates the possibility of multiple solutions and problems involving the deformation of a thin nonlinear elastic beam resting on a nonlinear elastic foundation.

To appear in Trans. IMACS Conf., Montreal, Que., Aug. 1982.

NUMERICAL METHODS FOR STIFF SYSTEMS OF
TWO-POINT BOUNDARY VALUE PROBLEMS

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ABSTRACT

We develop collocation methods for a class of singularly-perturbed two-point boundary value problems where the critical Jacobian has a hyperbolic splitting with a fixed number of stable and unstable eigenvalues. We use the asymptotic representation of the solution and a collocation method to construct an approximate solution. This solution can either be accepted or supplied as an initial guess to a two-point boundary value code, such as COLSYS, for further refinement. Our methods are applied to several nonlinear problems.

In preparation for an ICASE Report, NASA Langley Research Center, Hampton, VA and to be submitted to SIAM J. Sci. and Stat. Comput.

FOCUSING PROBLEMS FOR A NONLINEAR SCHRÖDINGER EQUATION

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ABSTRACT

We consider a cylindrically symmetric Schrödinger equation with a cubic nonlinearity. It is known that this equation has solutions that self-focus if the initial data is strong enough. We study this problem numerically using a self-adaptive finite element code and seek to determine (i) the quantitative nature of the solution as it focuses and (ii) whether the solution will still focus in the presence of a small amount of dissipation.

In preparation for Physica D.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) During the period covered by this report, the investigators continued their research on the development and application of numerical methods for singularly-perturbed (or stiff) boundary value problems for ordinary differential equations and initial-boundary value problems for partial differential equations. Results were obtained for collocation methods for vector systems of two-point boundary value problems and for adaptive grid finite element methods for partial differential equations. The investigators are applying their methods to several interesting physical problems, such as the deformation of nonlinear (CONTINUED)		

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ITEM #20, CONTINUED: elastic beams and a nonlinear Schrodinger equation which exhibits self focusing.

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